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EDUCATION

Listening for Common Ground in High School and Early Collegiate Mathematics

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Introduction

Solutions to pressing and complex social challenges require that we reach for common ground. Only through cooperation among people with a broad range of backgrounds and expertise can progress be made on issues as challenging as improving student success in mathematics. In this spirit, the AMS Committee on Education held a forum in May 2022 entitled *The Evolving Curriculum in High School and Early Undergraduate Mathematical Sciences Education*.[1] This article is a report on that forum by the authors listed above, who were among the organizers and presenters.

Fundamental premises

We took as a starting premise that the status quo is failing too many students, particularly students from underserved communities. Gaps in mathematics achievement between top and bottom quartiles of socioeconomic status are unlikely to close this century [HLP$^+$22]. Race continues to correlate with who experiences less access to grade-level mathematics, less access to experienced and skillful teachers, less meaningful engagement with mathematics, and as a result disproportionately low levels of success [Ber21] [Han14] [TNT18]. Ultimately, mathematically intensive professions suffer because the perspectives of missing demographics are underrepresented in mathematically intensive fields. As a discipline, and in collaboration with those with complementary expertise, we need to identify the foundational mathematics critical for all students to access the full range of postsecondary opportunities, and articulate the policies that will ensure that all students have equitable opportunities to learn and transition to those postsecondary opportunities.

Students from all backgrounds often end up repeating significant amounts of coursework completed in high school, while approximately one-fifth to one-third of students withdraw from or receive a D or F in precalculus and calculus [Joh22]. In mathematics departments, placement tests and policies to assess college readiness in math seem to condemn students to unsuccessfully repeating what they had already failed to learn. Many students never get a chance to see the purpose, flexibility, and beauty of mathematics until they get to advanced college courses—and most people don't make it that far. By the time students arrive in colleges and universities, mathematics departments struggle to attract and train majors, with a substantial barrier being the need to convince students that mathematics is about reasoning and understanding.

Many stakeholders agree that there is a need for more students to learn mathematics well and gain access to opportunities grounded in mathematics.[2] However, much work remains to determine how that can be achieved.

A focus on options for mathematics courses or course sequences

The broader mathematical community is already engaged in many review and revision efforts. These range from the call of the National Council of Teachers of Mathematics to "Catalyze Change" [NCTM18], to resources and programming from mathematical societies [MAA15] [CBMS19], to individual state systems engaged in reform. Through our May 2022 forum, we hoped to identify points of consensus among those from different backgrounds and perspectives. We discussed the topic of options for mathematics courses or course sequences at the high school and early undergraduate level with a focus on programs that were designed with equity in mind.

Focusing on these options encourages stakeholders to view high school and college mathematics not as two categories with a clear dividing line but rather a continuum. Such a focus also helps us improve the actual experiences of instructors and students moving through this space. Some sequences traverse through developmental mathematics, the collection of college courses that do not impart college credit. The mathematics addressed in these courses are sometimes referred to as "arithmetic", "early algebra", and "intermediate algebra".[3] The ideas in these courses are prerequisites for tackling college-level mathematics. The existence of developmental math reveals a truth that we cannot afford to ignore: many students arrive at college unable to reason about proportions, basic mathematical notation, algebraic operations, and more.

For the purposes of this article, we take college-level mathematics to refer to courses including: statistics, calculus, differential equations, linear algebra, or beyond. What is deemed "college-level" changes over time, and may differ by institution, with "college algebra" and "precalculus" often hovering nervously between the two levels. In fact, college algebra constitutes a fifth of the mathematics taught in two-year colleges and almost half of the mathematics taught at four-year colleges [CBMS15], and yet it appears to provide weak preparation for college calculus [SS16]. Real-world needs of industry and citizenry, as well as the needs of mathematically intensive fields, drive college curricula. We acknowledge the need for more consideration and alignment between the capacities students develop in our courses and the intellectual demands of modern society and future academic coursework. Fundamentally, college-level mathematics requires students to take mathematical ideas and reason with them, put them together, take them apart, and build upon them. Ultimately, we believe options for K–16 students should provide rich opportunities to learn mathematics, promote curiosity, ensure mobility, and produce graduates who have the mathematical capacities needed in our society.

Listening for potential areas of common ground

At the forum, Gail Burrill, Brian Conrad, Bill McCallum, Ricardo Moena, and Ji Son presented their perspectives on curriculum-related initiatives they help to lead. These perspectives are informed by their diverse experiences in teaching at high school and undergraduate levels, leading high school and university departments of mathematics, developing curricular materials, and orchestrating statewide and national initiatives for curricular change. They represent practitioners who participate in sustained conversations across disciplines and who work in a variety of contexts, from rural to urban to suburban. Their backgrounds vary across research in mathematics, mathematics education, and cognitive science.

During and after the forum, Henry Cohn, Yvonne Lai, Dev Sinha, Ji Son, and Kate Stevenson reviewed a recording of the session[4] for consensus across speakers. What follows is a list of areas where we found common ground.

We Have a Lot to Learn About Teaching Mathematical Applications at All Levels, Including at the Undergraduate Level

We want all students to have a deeper understanding of mathematics, whether for its own sake, or for applications that examine the world around us. If we wish to invite more students into mathematics, we must expand our vision of mathematical applications, and be open to collaboration with those in other disciplines who use mathematics, including hearing their perspectives on what students need. A broader view of applications can help students see the power of mathematics and prepare them for their potential careers—and it can also help them experience joy and wonder in mathematics.

As Brian learned in leading the redesign of Stanford University's Math 51, a course for general students that addresses high-dimensional linear algebra and multivariable optimization, there are applications that are incredibly interesting to students—and they are ones that he and the other mathematicians overhauling the course would have never known about, if not for their deliberate conversations with engineers, statisticians, and other science and technology faculty across their campus. For instance, though we as mathematics faculty may be aware of Markov chains being used in Google's now famous PageRank algorithm, or the use of quadratic forms in rigid-body mechanics, we may not all have known about the chain rule used in backpropagation in neural networks, the application of singular value decomposition to genetics, or the use of subspace projections in modern investment portfolio theory. Even if we are aware of some of these applications, we may not know how to describe them in terms that have integrity to the disciplines from which they come. Yet being able to communicate these applications is a way to invite students into mathematics. Brian reported that he and his colleagues discovered that students appreciated the content much more and enrollment in Math 51 grew vigorously while that of the analogous class from the school of engineering dropped precipitously. Similar success in infusing classes with applications has been seen on other campuses. Notably, Tyler J. Jarvis's April 2022 Inaugural AMS Education Lecture at the Joint Mathematics Meetings described the process that Brigham Young University followed to create a robust applied mathematics sub-major. Their first step was reform of their linear algebra course along similar lines, for example using orthogonal decomposition in digital filtering.

Collaboration with interdisciplinary colleagues can help mathematics faculty reimagine the mathematics that is possible to teach—or not teach. When meeting with colleagues while designing the new Math 51 course, a statistics colleague said to Brian, “You people aren’t still teaching row reduction, are you?” It was not until this exchange that Brian considered that there may be a way to teach linear algebra without so much emphasis on row reduction. As he commented in the forum, “You might be thinking, ‘How can you possibly teach linear algebra without row reduction?’” The answer is to have students think more about matrices and the structure of matrix algebra. This is more interesting both mathematically and with respect to its applications. Moreover, through discussions with other faculty, it became clear that a significant part of linear algebra content can be well-motivated and developed prior to discussing the non-linear optimization topics, whereas the non-linear optimization topics are better illuminated when preceded by the linear algebra content. This inversion of the usual order of topic development appears to serve students better, because they can do interesting mathematics applications immediately. Brian found collaboration with those in partner disciplines led to insights in linear algebra and multivariable optimization; we suspect there are insights to be had for many other service courses at the undergraduate level, as well as in courses at the secondary level.

In developing materials that incorporate applications, writers need to remember that applications and mathematical concepts are related but distinct. We cannot substitute one for the other; they are both needed. In Ji and Bill’s experiences writing curricula, they worried that applications, if not done well, could fail to deepen a student’s understanding of mathematics, and leave the impression of mathematics being a fragmented discipline in disconnected contexts. However a course addressing functions in the absence of modeling any phenomena or data would also not serve students well (e.g., leading to the misconception that functions are primarily used to solve for x or evaluating $f(x)$ at a particular value of x).

As mathematics faculty, we have an obligation to collaborate with our partner disciplines, both for the sake of our students, and for the sake of our profession. Our students deserve to see mathematics in a way that is interesting to them, and also in a way that sets them up for many potential future careers. In all sectors of US higher education, quantitative majors, minors, specializations, and certificates are flourishing. Examples include certificates in computational intelligence and linguistics, bachelor’s and associate’s degrees in data science, and master’s degrees in financial engineering [NASEM18]. These quantitative programs are offered by many academic disciplines, not just mathematics departments. However, math departments that incorporate such quantitative options appear to be booming (e.g., Macalester College, where approximately 10% of students major in math) and more generally, enrollments in upper division statistics offered by mathematics departments increased by 60% between 2010 and 2015 [CBMS15, Chap. 1]. To be stewards of our own profession, we must find ways to support students and uphold the integrity of mathematics that is taught. We are part of the safety net that ensures the quality and reliability of the mathematical curriculum.

All forum presenters believed that we must engage an ever more diverse student body in beautiful, relevant, and rigorous mathematics, and that it is possible for us to position ourselves to do so.

We Need to Articulate Mathematical Concepts, Skills, and Practices Across Educational Levels and Do So in Communication with Professionals Who Use Mathematics

Articulation came up in two important and interrelated ways during the forum. The first relates to identifying the foundational mathematical concepts, skills, and practices critical for students to access the full range of opportunities available in schools, colleges, and beyond. The second focuses on the policies, pedagogies, and support systems that will ensure that students have equitable opportunities to learn and to move fluidly between the institutions in our complex educational ecosystem.

High school mathematics can be reorganized to allow for deeper understanding of how these concepts fit together and for what purpose. Part of the reorganization called for is to question what students *do* with math. Too many students currently think of mathematics as solving equations with rules and strategies. For instance, spending time and attention adding three complex rational polynomial expressions by hand often means that students miss out on more important concepts such as division by 0 or losing a point of discontinuity. Ji suggested there may be ways of teaching elementary functions so that students use functions to model phenomena rather than using them to solve or evaluate with values of x. In Ji's experience, students begin asking questions such as: Are there functions that curve? Are there functions that can model a pattern that repeats? These kinds of questions motivate the notions of quadratic, polynomial, and trigonometric functions. The fundamental practice of mathematical modeling (e.g., Common Core Standards for Mathematical Practice 4) can help students understand something about how models work and are evaluated, in ways that can eventually generalize to even advanced models like machine learning. Such knowledge can also foster curiosity in learning more about functions.

Several speakers mentioned that contexts in the world around us are not the only interesting contexts. Mathematics can also be a context. What is essential is that students experience the coherence and wonder of pure mathematics, as well as the relevance of mathematics and computational techniques both to mathematics and to its applications. Identifying priorities and their potential structures will take time, as well as collaboration among mathematicians and a variety of professionals who use mathematics in their work–including K–12 mathematics teachers.

We need to examine the content and context of core topics, particularly in algebra, to ensure that students appreciate the modern relevance of this foundational mathematical materials, and how to convey the skills and concepts through curricular materials and instruction. For instance, Gail observed that terms such as "Algebra 2" can be a red herring in policy discussions, because the term ends up with more attention than the concepts that may be within it. In other words, over-reliance on terms such as "'Algebra 2" hides the hard and necessary work of drilling down into concepts, skills, and practices that students most need to learn, and how to set up students to learn them. By discussing broad courses by name rather than by specific content, we avoid the very challenging work of elaborating and motivating content, and thus rob ourselves of the very mechanism that would bring consensus.

In examining high school courses and sequences, we must also ensure that "pathways" in mathematics do not become troughs. Historically, lack of clarity about future opportunities has disproportionately harmed first-generation students and students from under-served communities. When options diverge early, they may inadvertently direct students away from algebraically intensive fields like economics, chemistry, physics, computer science, biology, data science, and many social sciences. As Brian noted, "A student may exit high school very interested in computer science, but they don't realize that college degrees in computer science often require calculus. So this student comes in, interested in a particular field, but their math background is specialized in a way that doesn't seem compatible with the requirements." To avoid unintended consequences—particularly for students of color and students from lower socio-economic backgrounds—mathematics curricula must be examined in the context of how mathematical concepts, skills, and practices intersect with other quantitative reasoning courses in secondary and post-secondary education.

Whatever system we are in, we should examine the extent to which students have accessible choices that meet their needs, the mechanisms used to place students in courses, the local availability of courses, and whether there is mobility across different options. CourseKata, a curriculum codesigned by Ji, Jim Stigler, and colleagues, and that addresses data science and statistics, is designed to fit into several pathways. It can be used directly after a course introducing variables and linear function, and it can be used before or after AP Statistics.

Once priorities for mathematics have been identified, all students should have the opportunity to see essentially the same content, when they are ready for it. They may not take it at the same time—one student may take it in eighth grade and another in eleventh grade—but no student should be locked out of any trajectory too early, and all students should be seeing content that sets them up for entering mathematically intensive fields at a university level.

We Must Acknowledge Teachers' Workload and the Need to Include in Any Curricular Proposals Realistic Support for Teachers

High school teachers, their peers teaching in lower grades, and untenured faculty at post-secondary institutions bear an extraordinary workload, particularly relative to the pay and respect they are afforded. All speakers and several participants forcefully pointed out the importance of ensuring that proposed changes to content and pedagogy are supported by realistic plans for how to support teachers enacting new curricular materials.

One thing that is clear from decades of curricular research is that teaching new materials is hard, and that curricular materials do not teach themselves. That is, an excellent teacher, with enough time and resources, can make good lessons out of poor curricular materials; and a teacher who lacks resources or training may teach a poor lesson out of wonderful curricular materials. The transformation of written curriculum materials into lessons depends on teachers' background, skills, resources, and the particular students in the classroom [SRS07]. When course content is new—whether because of a restructuring of concepts, a new perspective is taken, or another reason—teachers must do a lot of work to figure out how to reach students. When students come in with different backgrounds, teachers also must do work to adapt the materials to the students in front of them. As options for mathematics classes proliferate, we potentially change both the content that teachers have experience teaching, as well as the backgrounds of students who arrive at their classroom. These facts of life increase teachers' burden.

We need educative materials at all levels: that is, curricular materials that scaffold learning not just for students but also for teachers [DK05]. For instance, at the university level, Stanford's Math 51 materials could be considered educative in that they provide in great detail how the mathematics of various applications work. The detail is greater than what any particular student needs, but can also help instructors provide an accurate overview of those applications.

Educative materials can also suggest potential student thinking and possible pedagogical responses. They can provide alternative explanations for key concepts, or identify metaphors or analogies that students find helpful in understanding an idea. Educative materials might include various levels of scaffolding that can be deployed as necessary in the teaching context. For instance, in Illustrative Mathematics texts, an open-ended task might ask students to evaluate three potential approaches to a mathematical model, with an alternative, more scaffolded, version that offers students concrete suggestions for how to evaluate the potential approaches, then explicitly asks students to use the results of the suggestions to weigh the benefits and limitations of approaches. Teachers can choose the alternative that suits their classroom's needs best, learning as they go from considering the potential alternatives. These design ideas are not new, but designing and implementing them well for a variety of classrooms is a challenge, especially as we continue to carve out and experiment with potential pathways to and through the mathematical sciences.

To support the refinement of curricular materials and their use, developers and adopters need ways of gathering feedback. Some potential approaches include regular meetings about benchmarks. One example of this was reported by Ricardo, who described that in Ohio, there are teams of high school teachers and college faculty who meet regularly to discuss teaching and learning in the various courses available to high school students in Ohio. Others collect student clickstream, sentiment, and performance data, as the CourseKata team does. As Ji urged us to consider, after describing the CourseKata data collection process, "It doesn't matter how beautiful the words and figures are on a page, if the students aren't reading it." In other words, what developers have in mind may differ from what happens in real life, and one needs to do the hard work of examining the success of an approach tried. This effort is necessary for student success, and also to support those who teach the students.

When teaching new curricular materials, teachers can benefit from professional development directed at the use of these materials. One approach to professional development is an intensive start-up phase followed by continuous support similar to the professional development offered by the programs that Ji and Bill are involved with. For instance, CourseKata involves a six-week study group where instructors work collectively through prescribed lesson plans and data sets, followed up by continuing help through "office hours." Instructors may participate over multiple years. Ji reported that as instructors continue, they come with more sophisticated pedagogical reasoning, and more confidence in creating and sharing their own lesson plans. These approaches are consistent with the approaches taken by several courses developed for high school by faculty at The California State University [CSU22], which forthcoming research by Sherri Reed (UC Davis) indicates have positive impacts on acceptance and persistence in the California Community College System, the California State University System, and the University of California System.

For ongoing professional development during the school year, several speakers stressed that the bulk should be integrated into a teacher's normal work. It is important to note two separate issues here: time and relevance. With respect to time, we cannot depend on teachers, in high school or college, being "super human," dedicating their spare time to professional development. How does the parent juggling childcare on a minimal salary wedge this into their lives? This is the reality for too many high school teachers and untenured college faculty [Gon22]. The solution is to explicitly build time for professional learning into the work schedules of these professionals. That means decreasing time in the classroom and increasing costs. With respect to relevance, generic and sporadic professional development, which is often not aligned with the curriculum the teacher is using, has not proven effective at improving teacher performance [LFS+04] [TNT15]. Professional development for K-12 and university teachers appears to be more effective when tied to the goals for instructional practice in the classes assigned to the teacher, the expectations of excellence in their schools, and the culture of their communities [DHG17, Rec. 1] [TNT15] [JXY15, Rec. 4] [Ken16].

Stakeholders Need to Hear Each Other More Often, and the Way to Do That is Through Working Together

Altogether, the above points underscore the need for learning from those who have a variety of backgrounds and expertise. Courses must account for the possible directions and applications that the content can move toward. Potential pathways must afford mobility. Teachers need support in enacting new curricula. To design courses that engage students and open doors, we need high school teachers, college faculty, researchers, and industry professionals to hear each others' aims and constraints, and to support each others' efforts.

But we cannot just put people with different perspectives in a room and expect productive conversation to happen. For instance, we may need to be explicit about our perspectives, just as in Gail's advice to specify the content intended by terms such as "Algebra 2". Similarly, when using terms such as "data fluency", "data literacy", or "data science", we should press ourselves and each other to get down into specific concepts, procedures, and practices, and why these are at stake. Brian explained that in his conversations to redesign Stanford's Math 51, and also in conversations about mathematics education more broadly, he heard the refrain, "Where is it going?" The only way to expand our notions of where mathematics might lead is to talk to those who use mathematics.

Once we identify consequential ideas, we need to articulate working definitions that represent consensus. Ricardo relayed the story of creating mathematical options in Ohio, and pushback from mathematics faculty who wondered whether the options would be less mathematically rigorous than the status quo. Mathematical rigor was at stake. To understand and move forward from this concern, high school teachers and collegiate faculty working on Ohio's high school options came to a consensus on a definition of mathematical rigor: "Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate how, when, and why their procedure works and why it is appropriate. Students can answer the question, 'How do we know?'" This definition, formed through communication across parties, was then a tool for evaluating the course syllabi and content. The lesson here is not the particular definition of "rigor"; there may be other ways to conceive of or phrase a definition of rigor. The lesson is that one way of finding consensus across stakeholders who work in different contexts is to come together to craft definitions that serve a common purpose. This is the process Ohio's Department of Education followed in developing its definition of rigor as well as its mathematical options.[5] For the people in Ohio who came together to formulate this definition of rigor, the definition represents not only its literal meaning, but also the debates and discussion that went into the definition. This story also shows the need for and power of ongoing conversation among stakeholders: this discussion needed to unfold over time.

As much as we ask, "Where is it going?", we might also ask, "Where is it coming from?" Here too, negotiating different perspectives is important. As Bill commented after being asked how curriculum developers sequence content and determine prerequisites, "There's the 'logical' structure of mathematics and the 'pedagogical' structure of the students' minds." One of Illustrative Mathematics' design principles is to render the logical structure visible while sequencing the concepts in an order that makes sense pedagogically.

And then there is the question of, "How do we sustain this over time, especially across communities?" As mentioned above, Ohio uses teams of faculty from high schools, two-year colleges and universities to continuously support and review programs. Ricardo described how each team is regionally focused, is stable in its membership, and meets biweekly to discuss progress, suggest pedagogies, and debate relevance. Thus, while the framework of Ohio's four options for the high school curriculum ensures statewide continuity, these stable local teams keep the curriculum discussions manageable and relevant. As Ricardo pointed out, "Someone up by Lake Erie doesn't have the same reality as someone who lives in southwest Ohio where it is coal country." One challenge Ohio still faces is creating and sustaining stable local teams across vast rural areas.

Another challenge is the teacher's own context. Gail illustrated a range of possible situations. A friend of hers recently took a job as a teacher on tribal lands and is now the first mathematics specialist the school has ever hired. There are teachers who teach three subjects in the same hour in the same classroom, or who teach in a one-room schoolhouse spanning middle and high school grades. These examples underscore the nation's struggle to equitably educate students in rural communities and more generally to do so in contexts where progress depends not only on teachers and students but also the broader community. On the other hand, the examples also point to opportunities in which the right investments could effect powerful change like those that grow teacher training programs and support existing math teachers in their communities.

In summary, stakeholders need to hear each other more often, and the way to do that is through working together. Ricardo's work in Ohio involved sustained conversations between two-year institutions, four-year institutions, and high school teachers. Brian and colleagues' project to renew Stanford Math 51 took a team of 6–7 dedicated, tenured math faculty (including the department chair and a Fields medalist) working over two years on top of their normal duties. CourseKata combines the expertise of faculty across a variety of institutions from high schools to community colleges to comprehensive universities to research-intensive institutions, as well as the expertise of faculty in mathematics departments, data analytics, and learning scientists. Brian observed that the resulting textbook "had all sorts of interesting examples that would not have been there except for talking to colleagues in other departments." The curricular developments of Ricardo, Bill, and Ji require communication not only across educational segments but also with community members, such as students, their families, high school and college counselors, and university and high school administrators. What each of these innovators mentioned was that in doing the work they achieved more than just the task at hand. They and their collaborators deepened their understanding of one another's expertise and context and formed personal connections. These were critical to jump starting later work, lowering the barriers to making significant progress in student success.

What have we learned from these cases? Revising mathematics curricula is hard work, both because the content is challenging and because there are real disagreements on priorities. There is no better way for disagreeing parties to come to agreement than for them to work together on a clearly defined project. Through that work, bit by bit, the path to progress opens up. It is critical that we identify and define projects on which stakeholders can work together.

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